tray fall on the single line in Figure 1 labelled "uniform vapor distribution."

The following condensed table gives the efficiency ratio for the limiting vapor distributions at various values of $E\lambda$:

$E\lambda$	$(E_{mv}/E)^0$	$(E_{mv}/E)^1$
0.5	1.30	1.37
1.0	1.70	2.00
1.5	2.32	2.96
2.0	3.20	4.42

0—Uniform vapor distribution
1—First row of caps inactive

As in Lewis' analysis the effect is more pronounced at higher values of $E\lambda$, and as would be expected the effect increases monotonically with the amount of imbalance in the vapor flow. The error in the predicted tray efficiency can be as high as 20 or 30% if the vapor flow is sufficiently nonuniform. Figure 1 can be used to determine the efficiency ratio for combined vapor flow and concentration gradients.

ACKNOWLEDGMENT

The author would like to thank Dr. Harold S. Kemp of the E. I. du Pont Experimental Station and Dr. Robert L. Pigford of the University of Delaware for helpful criticism and suggestions.

NOTATION

E = Murphree point efficiency
E_{mV} = Murphree tray efficiency
g = point value of the vapor flow rate
$\overline{G}$ = flow rate averaged over entire tray
L = liquid flow rate
m = slope of equilibrium line, $y^* = mx + b$
w = distance along the tray, as a fraction of the total
y = vapor composition
y_{n-1} = vapor composition under the tray
y_0 = vapor composition at exit of tray ($w = 1$)
y^* = vapor composition in equilibrium with liquid at any point
$\overline{y}$ = vapor composition averaged over entire tray
z = fractional imbalance in vapor flow rate at end of tray (for linear vapor-flow gradient)
$= \dfrac{\overline{G} - g}{\overline{G}}$ evaluated at $w = 0$
λ = $m\overline{G}/L$

LITERATURE CITED

1. Lewis, W. K. Jr., *Ind. Eng. Chem.*, **28**, 399-402 (1936).

A Note on Transport to Spheres in Stokes Flow

S. K. FRIEDLANDER

The Johns Hopkins University, Baltimore, Maryland

Mass and heat transfer to spheres at Reynolds numbers sufficiently large for a velocity boundary layer to exist have received a considerable amount of attention in the literature. Experimental data have been obtained over wide ranges of N_{Re} and N_{Sc}, and a satisfactory theory has been developed for calculating the transfer rate at least up to the point of separation of the velocity boundary layer.

Mass transfer at Reynolds numbers below about 1, the range of Stokes flow, has received less attention This range is of importance for drops smaller than about 100 μ falling through air, bubbles smaller than about 100 μ rising through water, and for particles and drops moving through liquids. Transport in the Stokes flow region is relatively difficult to study experimentally, and only a few data are reported in the literature. Theory indicates that unlike the result for high N_{Re}, the Sherwood number (the Nusselt number for mass transport) is a function only of the Péclet number N_{Pe} and not of N_{Re} and N_{Sc} separately.

An approximate theoretical curve for the Sherwood number over the entire Peclet number range has been obtained by the author of this note (*1*) who assumed the existence of a concentration boundary layer and used the von Karman integral method. For $N_{Pe} = 0.3$, 1, 3, and 10 Yuge (*6*) has calculated what are presumably more accurate values for the Sherwood number by a numerical solution of the complete diffusion equation. A better curve than that given in (*1*) can be obtained by the use of the Yuge numerical data for low N_{Pe} and the Levich-Lighthill (*3, 4*) thin boundary-layer method for $N_{Pe} \to \infty$.

For thin concentration boundary layers it is necessary to take into account only the first term in an expansion of the velocity in the region near the wall when the equation of convective diffusion is solved. For the region close to the surface of the sphere the diffusion equation can be written (*2*) as

$$u_1 \frac{\partial c_1}{\partial x_1} + v_1 \frac{\partial c_1}{\partial y_1} = \frac{2}{N_{Pe}} \frac{\partial^2 c_1}{\partial y_1^2} \quad (1)$$

The first terms in the expansion of the Stokes velocities give for u_1 and v_1

$$u_1 = (3/2)\, y_1 \sin x_1 \qquad (2)$$
$$v_1 = -(3/2)\, y_1^2 \cos x_1$$

Let $\eta = \alpha^{-1/3}\, y_1 \sin x_1 / \left(\int_0^{x_1} \sin^2 x'_1\, dx'_1\right)$ and assume $c = f(\eta)$. Substituting in (1) one gets

$$\frac{1}{N_{Pe}} \frac{d^2 c_1}{d\eta^2} + \frac{\alpha}{4} \eta^2 \frac{dc_1}{d\eta} = 0 \quad (3)$$

The solution which satisfies the boundary condition $c_1 = 1$ at $\eta = 0$ and $c_1 = 0$ at $\eta = \infty$ is

$$c_1 = 1 - \frac{\int_0^{\eta} e^{-\alpha N_{Pe} \eta'^3/12} dn'}{\int_0^{\infty} e^{-\alpha N_{Pe} \eta'^3/12} dn'} \quad (4)$$

The Sherwood number is given by

$$N_{Sh} = -\int_0^{\pi} \left(\frac{\partial c_1}{\partial y_1}\right)_{y_1=0} \sin x_1\, dx_1$$
$$= -\alpha^{-1/3} \left(\frac{dc_1}{d\eta}\right)_{\eta=0} \int_0^{\pi} \frac{\sin^2 x'_1\, dx'_1}{\left(\int_0^{x_1'} \sin^2 x_1''\, dx_1''\right)} \quad (5)$$

From Equation (4)

$$\left(\frac{dc_1}{d\eta}\right)_{\eta=0} = -\frac{(3/2)^{2/3}}{\Gamma(1/3)} (\alpha N_{Pe})^{1/3} \quad (6)$$

Hence (5) becomes*

* Using the thin concentration boundary-layer theory Natanson (5) finds for flow normal to a single cylinder:

$$N_{SH} = 1.17\, N_{Pe}^{1/3}/2(2 - \ln N_{RE})$$

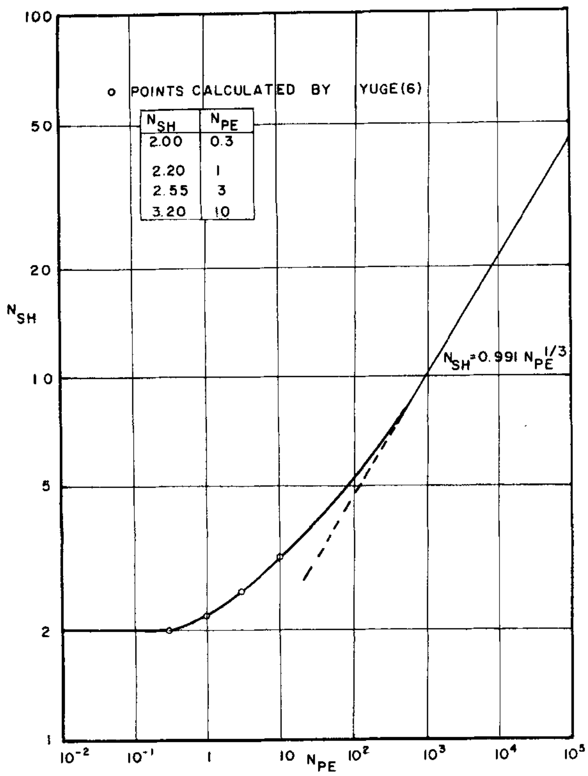

Fig. 1. Transport to spheres in Stokes flow.

$$N_{Sh} = \frac{(3\pi)^{2/3} \qquad N_{Pe}^{1/3}}{4(2)^{1/3}\ \Gamma(4/3)}$$
$$= 0.991\, N_{Pe}^{1/3} \qquad (7)$$

Levich finds for the numerical value of the constant 0.997 (*3*). The complete N_{Sh} vs. N_{Pe} curve is shown in Figure 1 based on the values calculated by Yuge and Equation (7). The thin boundary-layer approximation is probably satisfactory for $N_{Pe} > 10^2$, but it would be desirable to have more numerical calculations for the range $10^3 > N_{Pe} > 10$. The Sherwood number is very close to 2 for $N_{Pe} < 0.3$. It should be noted that the approximate curve given in (*1*) falls quite close to Figure 1.

ACKNOWLEDGMENT

This work was supported by NSF Grant G 5079.

NOTATION

- a = radius of sphere (or cylinder), cm.
- c_1 = ratio of point concentration to surface concentration, dimensionless
- D = diffusion coefficient, sq.cm./sec.
- k = mass transfer coefficient, cm./sec.
- N_{Pe} = Péclet number, $2aU/D$, dimensionless
- N_{Re} = Reynolds number, $2aU/\nu$, dimensionless
- N_{Sc} = Schmidt number, ν_{Nu}/D, dimensionless
- N_{Sh} = Sherwood number, $2ak/D$, dimensionless
- u_1 = ratio of point velocity in x direction to U, dimensionless
- U = fluid velocity at infinity, cm./sec.
- v_1 = ratio of point velocity in y direction to U, dimensionless
- x_1 = ratio of distance measured along a meridian from the stagnation point to a, dimensionless
- y_1 = ratio of distance normal to surface to a, dimensionless

Greek Letters

- α = arbitrary constant, dimensionless
- η = boundary layer variable, dimensionless
- ν = kinematic viscosity, sq.cm./sec.

primed quantities denote dummy variables

LITERATURE CITED

1. Friedlander, S. K., *A.I.Ch.E. Journal*, 3, 43 (1957).
2. Frossling, N., *Beitrage zur Geophysik*, **52**, 170 (1938).
3. Levich, V. G., "Physicochemical Hydrodynamics," Moscow (1952); as reported in (*5*) and in a personal communication with G. L. Natanson, Karpov Institute for Physical Chemistry, Moscow.
4. Lighthill, M. J., *Proc. Roy. Soc.*, **A202**, 359 (1950).
5. Natanson, G. L., *Doklady Akad. Nauk. S.S.S.R.*, **112**, 100 (1957).
6. Yuge, T., *Rept. Inst. High Speed Mechanics of Tohoku Univ.*, **6**, 143 (1956).

Mass Transfer in Stirred Vessels

R. B. OLNEY

Shell Development Company, Emeryville, California

Treybal (*11*) has analyzed certain liquid-liquid mass transfer data obtained in baffled vessels equipped with flat-blade turbines (*3, 9*). For his model he used the Calderbank-Korchinski (*2*) approximation for diffusion in the drop phase, while the coefficient for the outside liquid film was estimated from the equation of Johnson and Huang (*5*) for dissolution of solid rings fixed in place in a baffled, stirred vessel. Other over-all transfer data (*9*) were obtained with the system kerosene-water-butylamine (solute) with marine propellers and spiral turbines in a 15-in. diameter baffled vessel, but the results were not included in Treybal's analysis. It seems worthwhile therefore to show a similar analysis of these data.

One will use a still simpler idealization in which one neglects the continuous phase resistance and assumes that the over-all transfer rate is limited solely by molecular diffusion in the drops. One assumes spherical drops, so that $a = 6\phi/d$. Further one assumes that the Kolmogorov (*8*) or Hinze (*4*) mechanism for maximum stable drop size in an isotropic homogeneous tur-